Certification Examinations for Oklahoma Educators™

Oklahoma Subject Area Tests™

STUDY GUIDE

025 Middle Level/Intermediate Mathematics

Oklahoma Commission for Teacher Preparation

OK-SG-FLD025-04

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STUDY GUIDE INTRODUCTION AND GENERAL INFORMATION ABOUT THE CERTIFICATION EXAMINATIONS FOR OKLAHOMA EDUCATORS

The first two sections of the study guide are available in a separate PDF file. Click the link below to view or print these sections.

Study Guide Introduction and General Information About the Certification Examinations for Oklahoma Educators

FIELD-SPECIFIC INFORMATION

• Test Competencies
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INTRODUCTION

This section includes a list of the test competencies, as well as a set of practice selected-response (multiple-choice) questions and one or more practice constructed-response assignments (if applicable), for the test field included in this study guide.

Test Competencies

The test competencies are broad, conceptual statements that reflect the subject-matter skills, knowledge, and understanding an entry-level educator needs to teach effectively in Oklahoma public schools. The list of test competencies for each test field represents the **only** source of information about what a specific test will cover and therefore should be reviewed carefully.

The descriptive statements that follow the competencies are included to provide examples of possible content covered by each competency. These descriptive statements are neither exhaustive nor exclusionary.

Practice Test Questions

The practice selected-response questions and any practice constructed-response assignments included in this section are designed to give you an introduction to the nature of the questions included in this OSAT test field. The practice test questions represent the various types of questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or to predict your performance on the test as a whole.

To help you prepare for your OSAT, each practice selected-response question in this section is preceded by the competency it measures and followed by a brief explanation of the correct response. On the actual test, the competencies, correct responses, and explanations will **not** be given.

If the test field included in this guide has a constructed-response assignment, a sample response is provided immediately following the practice constructed-response assignment. The sample response in this guide is for illustrative purposes only. Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

A description of the process that is used for scoring the constructed-response assignment is provided in addition to the OSAT performance characteristics and score scale.

When you are finished with the practice test questions, you may wish to go back and review the entire list of test competencies and descriptive statements for your test field.

SUBAREAS:

I. Mathematical Processes and Number Sense
II. Relations, Functions, and Algebra
III. Measurement and Geometry
IV. Probability, Statistics, and Discrete Mathematics

SUBAREA I—MATHEMATICAL PROCESSES AND NUMBER SENSE

Competency 0001

Understand mathematical problem solving and the connections between and among the fields of mathematics and other disciplines.

The following topics are examples of content that may be covered under this competency.

Analyze and apply a variety of problem-solving strategies to contexts within and outside mathematics.

Select and use appropriate manipulatives and technological tools (e.g., spreadsheets, graphing utilities) to solve problems.

Recognize and apply connections among mathematical concepts in different fields (e.g., algebra, geometry, probability) and be able to apply mathematics in a real-world setting.

Demonstrate knowledge of the historical development of mathematics, including contributions from diverse cultures.

Competency 0002

Understand the principles and processes of mathematical reasoning.

The following topics are examples of content that may be covered under this competency.

Construct and evaluate mathematical conjectures, arguments, and informal proofs.

Apply inductive and deductive reasoning to solve problems.

Use counterexamples to evaluate arguments and disprove suppositions.

Apply proportional and spatial reasoning (e.g., using ratios or geometric concepts) to solve problems.

Competency 0003

Understand and communicate mathematical concepts, symbols, and terminology.

The following topics are examples of content that may be covered under this competency.

Convert everyday language into mathematical language, notation, and symbols, and vice versa.

Analyze, use, and perform conversions among algebraic, graphic, pictorial, and other modes of presenting and modeling mathematical concepts and relationships.

Deduce the assumptions inherent in a given mathematical statement, expression, or definition.

Evaluate the mathematical thinking and strategies of others.

Competency 0004

Understand number theory and the principles and properties of the complex number system (i.e., real and imaginary numbers).

The following topics are examples of content that may be covered under this competency.

Apply the operations of complex numbers (e.g., integers, fractions, decimals, percents, rational exponents) in problem-solving situations.

Analyze and apply the properties of complex numbers (e.g., associative, distributive, commutative).

Represent and interpret numbers in exponential and scientific notation.

Understand the fundamentals of number theory (e.g., prime numbers, divisibility, order of operations).

Understand the hierarchy of the complex number system and its reclassification into various subsets.

SUBAREA II—RELATIONS, FUNCTIONS, AND ALGEBRA

Competency 0005

Understand mathematical patterns and use them to solve problems.

The following topics are examples of content that may be covered under this competency.

Identify and extend a variety of numerical and geometric patterns.

Analyze and generalize sequences and series (e.g., arithmetic, geometric) and use them to model and solve problems.

Analyze and develop algebraic generalizations of different types of patterns (e.g., recursive, exponential).

Use patterns and functions to represent and solve problems.

Competency 0006

Understand the principles and properties of algebraic relations and functions, including inverses and compositions.

The following topics are examples of content that may be covered under this competency.

Analyze relationships among different representations (e.g., tabular, algebraic, graphic) of relations and functions.

Analyze relations and functions and their graphs in terms of domain, range, symmetry, intercepts, maxima, and minima.

Distinguish between relations and functions.

Determine the effects of transformations such as $f(x + k)$, $f(x) + k$, and $kf(x)$ on the graph of the relation or function $f(x)$.

Competency 0007

Understand the properties of linear functions and relations.

The following topics are examples of content that may be covered under this competency.

Determine the equation of a line and interpret the slope and intercept in mathematical and everyday contexts.

Develop the equation of a line on the basis of different types of information (e.g., two points on the line, the slope and one point on the line).

Model and solve problems involving linear equations and inequalities using algebraic and graphic techniques.

Solve systems of linear equations and inequalities in mathematical and everyday contexts using a variety of techniques (e.g., substitution, graphing, linear combination, matrices).

Competency 0008

Understand the properties of quadratic and higher-order polynomial functions and relations.

The following topics are examples of content that may be covered under this competency.

Analyze relationships among different representations of quadratic and higher-order polynomial functions (e.g., tabular, algebraic, graphic).

Model and solve problems involving quadratic and higher-order polynomial equations and inequalities using a variety of techniques (e.g., completing the square, factoring, graphing, quadratic formula).

Analyze the roots of quadratic and higher-order polynomial equations.

Analyze and use the equations and graphs of conic sections.

Competency 0009

Understand the principles and properties of rational, absolute value, exponential, and logarithmic functions.

The following topics are examples of content that may be covered under this competency.

Manipulate and simplify rational, absolute value, exponential, and logarithmic expressions.

Describe and analyze characteristics of rational, absolute value, exponential, and logarithmic functions and their graphs (e.g., asymptotes).

Convert algebraic representations of rational, absolute value, exponential, and logarithmic functions into graphic representations, and vice versa.

Model and solve problems involving rational, absolute value, exponential, and logarithmic equations in mathematical and everyday contexts.

SUBAREA III—MEASUREMENT AND GEOMETRY

Competency 0010

Understand principles and procedures related to measurement.

The following topics are examples of content that may be covered under this competency.

Apply appropriate techniques, tools, and units to determine measurements.

Apply formulas to find measures (e.g., angles, length, perimeter, area, volume) of a variety of two- and three-dimensional figures.

Solve problems involving derived units (e.g., density, pressure, rates of change).

Compare and convert measurements within customary and metric measurement systems.

Competency 0011

Understand the principles and properties of Euclidean geometry in two and three dimensions.

The following topics are examples of content that may be covered under this competency.

Use the properties of lines (e.g., parallel, perpendicular) and angles (e.g., supplementary, vertical) to characterize geometric relationships and solve problems.

Apply the principles of similarity and congruence to solve problems involving two- and three-dimensional figures.

Apply the properties of circles (e.g., intersecting chords and secants) and polygons (e.g., numbers and lengths of sides, measures of angles) to analyze and solve problems.

Use principles and theorems of geometry to construct and evaluate informal proofs.

Competency 0012

Understand the principles and properties of coordinate and transformational geometries.

The following topics are examples of content that may be covered under this competency.

Apply geometric concepts (e.g., distance, midpoint, slope) to model and solve problems.

Apply the geometric concepts of parallel and perpendicular lines to model and solve problems.

Use two- and three-dimensional coordinate systems to represent and analyze geometric figures.

Analyze and apply geometric transformations (e.g., translations, reflections, dilations, rotations).

Competency 0013

Understand right triangle trigonometry and the conceptual foundations of calculus.

The following topics are examples of content that may be covered under this competency.

Use the sine, cosine, and tangent ratios in right triangles to solve problems.

Apply the concept of limits to algebraic functions and their graphs.

Relate the concept of the derivative to instantaneous rate of change and the concept of the slope of the line tangent to a curve.

Relate the concept of the integral to the area under a curve.

SUBAREA IV—PROBABILITY, STATISTICS, AND DISCRETE MATHEMATICS

Competency 0014

Understand the principles, properties, and techniques of probability.

The following topics are examples of content that may be covered under this competency.

Demonstrate knowledge of probabilistic events and their characteristics (e.g., conditional, independent, mutually exclusive).

Solve problems using the techniques of probability (e.g., addition and multiplication rules).

Use and interpret graphic representations of probabilities (e.g., tables, Venn diagrams, tree diagrams, frequency graphs, the normal curve).

Analyze and apply the properties of normal probability distributions to model and solve problems.

Competency 0015

Understand the principles, properties, and techniques of statistics.

The following topics are examples of content that may be covered under this competency.

Determine random sampling techniques to collect representative data.

Identify and use data in a variety of graphic formats (e.g., charts, bar graphs, circle graphs, stem-and-leaf plots, histograms, scatter plots, line of best fit).

Determine, analyze, and interpret measures of central tendency (e.g., mean, median) and dispersion (e.g., standard deviation).

Evaluate statistical claims, inferences, and predictions based on a set of data (e.g., analyzing sampling techniques, interpreting statistical measures).

Competency 0016

Understand the principles of discrete mathematics.

The following topics are examples of content that may be covered under this competency.

Apply various counting strategies (e.g., permutations, combinations, factorials) to problem-solving situations.

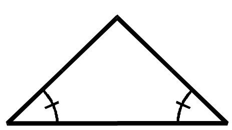
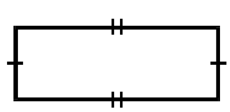
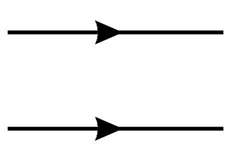
Analyze recurrence relations (e.g., Fibonacci sequence, triangular numbers) and use them to model and solve problems in mathematics and other disciplines.

Apply the basic elements of discrete mathematics (e.g., finite graphs, trees) to model everyday problems.

Identify potential applications of discrete mathematics (e.g., set theory, graph theory) across the curriculum.

Demonstrate a knowledge of matrices and their operations.

Definitions and Formulas for Middle Level/Intermediate Mathematics

Notation	Description
$a \rightarrow b$	*a* implies *b*
$a \leftrightarrow b$	*a* if and only if *b*
$a \wedge b$	*a* and *b*
$a \vee b$	*a* or *b*
$\sim a$	not *a*
$A \cup B$	*A* union *B*
$A \cap B$	*A* intersect *B*
$\overline{A}$	complement of *A*
U	universal set
$\{ \}$	empty set
$i = \sqrt{-1}$	imaginary unit
$\overline{z}$	complex conjugate of *z*
A^{-1}	inverse of matrix *A*
$\vec{\boldsymbol{v}}$	vector *v*
$\sim$	is similar to
$\cong$	is congruent to
	congruent angles
	congruent sides
	parallel lines

Formula	Description
$V = \frac{1}{3}Bh$	volume of a right cone and a pyramid
$A = 4\pi r^2$	surface area of a sphere
$V = \frac{4}{3}\pi r^3$	volume of a sphere
$S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$	sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	sum of a geometric series
$\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r}, \lvert r \rvert < 1$	sum of an infinite geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	slope
$y = ax^2 + bx + c$	parabola
$s^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}$	variance
$s = r\theta$	arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	quadratic formula
${}_nC_r = \frac{n!}{r!(n - r)!}$	combinations
${}_nP_r = \frac{n!}{(n - r)!}$	permutations

(continued on next page)

Formula	Description
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	sine of θ in a right triangle
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	cosine of θ in a right triangle
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	tangent of θ in a right triangle

END OF DEFINITIONS AND FORMULAS

All examinees taking the Middle Level/Intermediate Mathematics OSAT will be provided with a scientific calculator with functions that include the following: addition, subtraction, multiplication, division, square root, percent, sine, cosine, tangent, exponents, and logarithms. Please see the current CEOE registration bulletin for information regarding the brand and model of calculator that will be supplied. **You may NOT bring your own calculator to the test.**

Practice Selected-Response Questions

Competency 0001

Understand mathematical problem solving and the connections between and among the fields of mathematics and other disciplines.

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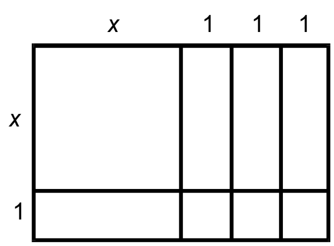
1. **Use the diagram below to answer the question that follows.**



Algebra tiles are arranged as shown in the diagram above. This arrangement of algebra tiles could represent which of the following expressions?

A. $(x + 1)(x + 1)^3$

B. $x^2 + 4x + 3$

C. $(x^2 + 3)(x^2 + 1)$

D. $2x^2 + x + 3$

Correct Response: B. A set of algebra tiles usually consists of large squares that measure x by x, rectangles that measure x by 1, and small unit squares that measure 1 by 1. The area of these squares can be used to represent quadratic expressions since the area of a large square equals x^2, the area of a rectangle equals x, and the area of a unit square equals 1. In the diagram given, there are one large square, four rectangles, and three unit squares. Hence, the tiles represent the quadratic expression $x^2 + 4x + 3$. This is also equivalent to $(x + 3)(x + 1)$, the product of the side lengths of the given figure.

Competency 0002

Understand the principles and processes of mathematical reasoning.

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2. For which of the following does $n = 6$ provide a counterexample?

   A. If n is divisible by 2, then $n^2 - 1$ is divisible by 5.

   B. If n is divisible by 3, then $n^2 - 1$ is divisible by 4.

   C. If n is divisible by 4, then $n^2 - 1$ is divisible by 5.

   D. If n is divisible by 5, then $n^2 - 1$ is divisible by 4.

Correct Response: B. A counterexample is an example that proves a conditional, or if-then, statement to be false. In other words, it shows that even though the condition of the statement is met, the conclusion is not true. Since 6 is divisible by 3, the condition is met. However, since $6^2 - 1$ is not divisible by 4, the conclusion in response B is false. Thus the statement $n = 6$ is a counterexample to the statement in response B.

Competency 0005

Understand mathematical patterns and use them to solve problems.

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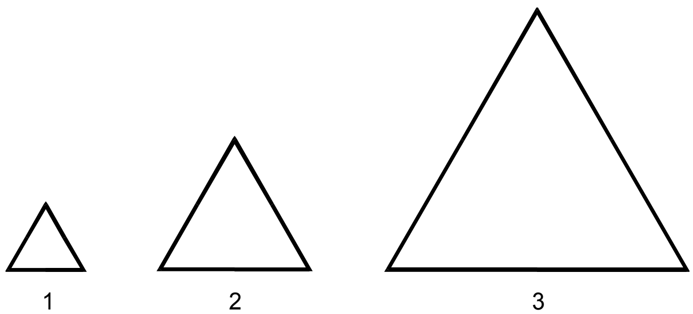
3. **Use the diagram below to answer the question that follows.**



The first three triangles in a sequence of equilateral triangles have areas $A_1 = \frac{\sqrt{3}}{4}$, $A_2 = \sqrt{3}$, and $A_3 = 4\sqrt{3}$. What is the ratio of A_6 to A_1?

A. 256

B. $256\sqrt{3}$

C. $512\sqrt{3}$

D. 1024

Correct Response: D. The ratio of the area of the second triangle in the sequence to the area of the first triangle is $\frac{A_2}{A_1} = \sqrt{3} \div \frac{\sqrt{3}}{4} = 4$. The ratio of the third triangle in the sequence to that of the second triangle is $\frac{A_3}{A_2} = 4\sqrt{3} \div \sqrt{3} = 4$. Assuming that this pattern continues throughout the sequence, then $A_4 = 4A_3 = 4(4\sqrt{3}) = 16\sqrt{3}$, $A_5 = 4A_4 = 4(16\sqrt{3}) = 64\sqrt{3}$, and $A_6 = 4A_5 = 4(64\sqrt{3}) = 256\sqrt{3}$.

Thus, the ratio of A_6 to $A_1 = 256\sqrt{3} \div \frac{\sqrt{3}}{4} = 1024$.

Competency 0006

Understand the principles and properties of algebraic relations and functions, including inverses and compositions.

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4. How does the graph of $f(x) = (x - a)^2 + 2(x - a) - 6$ compare with the graph of $g(x) = x^2 + 2x - 6$, given that $a > 0$?

   A. The graph of $f(x)$ is shifted a units to the right.

   B. The graph of $f(x)$ is shifted a units to the left.

   C. The slope of $f(x)$ is decreased by a units.

   D. The graph of $f(x)$ is shifted a units down.

Correct Response: A. Inspection of the two functions shows that $f(x)$ can be obtained from $g(x)$ by substituting $x - a$ for x in $g(x)$, or $f(x) = g(x - a)$. This means that the graph of $f(x)$ is a horizontal translation of the graph of $g(x)$, in this case a units to the right. For example if $a = 3$, the graph of $f(x) = (x - 3)^2 + 2(x - 3) - 6$ is identical to the graph of $g(x) = x^2 + 2x - 6$ shifted to the right 3 units.

Competency 0008

Understand the properties of quadratic and higher-order polynomial functions and relations.

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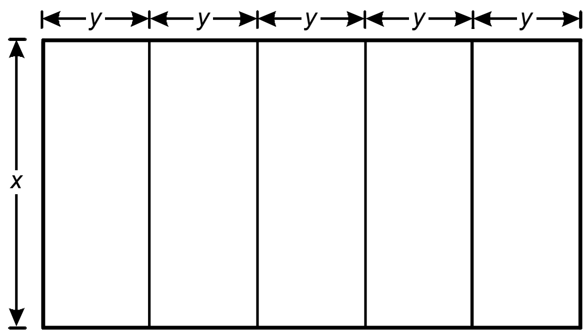
5. **Use the diagram below to answer the question that follows.**



A dog kennel owner plans to build five adjacent rectangular running pens out of 150 meters of fencing. If each pen measures x meters by y meters, with a total area of 468 square meters for the five pens, which of the following quadratic equations can be used to determine the value of x?

A. $6x^2 - 150x + 468 = 0$

B. $3x^2 - 75x + 468 = 0$

C. $x^2 - 75x + 468 = 0$

D. $x^2 - 150x + 468 = 0$

Correct Response: B. To build the dog pens as described, the dog kennel owner would need 10 pieces of fencing y meters long (5 pieces for each end) and 6 pieces x meters long. Therefore, $10y + 6x = 150$. Since the total area is the product of the length and the width, $468 = x(5y)$. The first equation is equivalent to $5y = 75 - 3x$. Substituting this equation into the expression for the area results in $468 = x(75 - 3x)$, which leads to the quadratic equation $3x^2 - 75x + 468 = 0$.

Competency 0010

Understand principles and procedures related to measurement.

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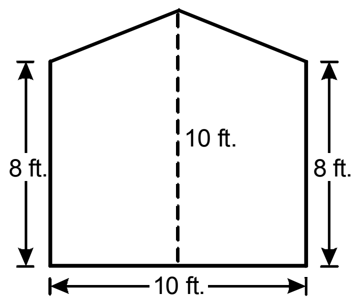
6. **Use the diagram below to answer the question that follows.**



Max plans to paint the exterior walls of his garage. The garage has a pentagonal front and back and rectangular sides. The diagram above shows the front of the garage. The sides are 8-foot-by-15-foot rectangles. One gallon of paint costs $18.50 and covers 350 square feet. Assuming he will need two coats of paint, how much will Max have to spend on paint?

A. $18.50

B. $37.00

C. $55.50

D. $74.00

Correct Response: C. Since each of the two sides of the garage is a rectangle with a length of 15 feet and a width of 8 feet, the area of each side is 120 square feet. The front and back of the garage are each a rectangle with a length of 10 feet and a width of 8 feet plus a triangle with a base of 10 feet and a height of 2 feet, so the areas of the front and back are 90 square feet each. The total area of the garage (not including the roof) is thus 420 square feet. Since two coats of paint are needed, the total area to be painted is 840 square feet. Because one gallon of paint covers 350 square feet and $\frac{840}{350} = 2.4$, three gallons of paint are needed, and the cost of three gallons of paint is $3(\$18.50) = \$55.50$.

Competency 0011

Understand the principles and properties of Euclidean geometry in two and three dimensions.

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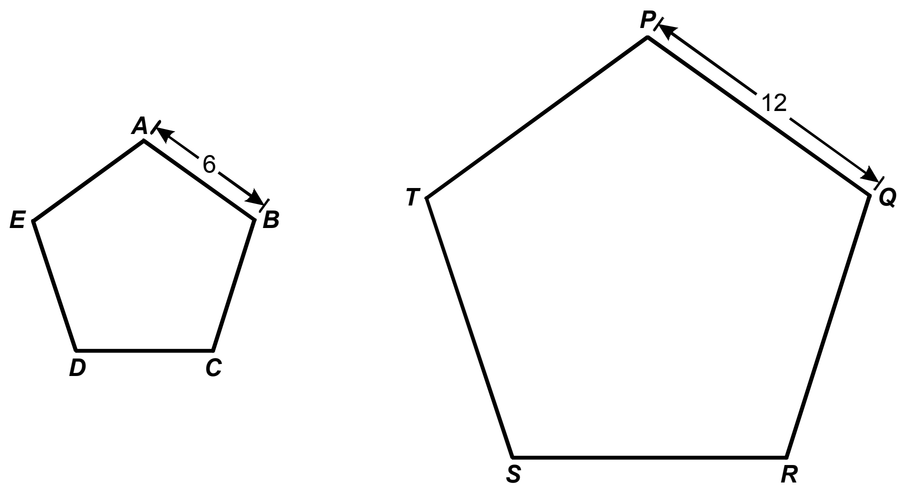
7. **Use the diagrams below to answer the question that follows.**



If $ABCDE \sim PQRST$, what is the ratio of the area of $ABCDE$ to the area of $PQRST$?

A. 1:9

B. 1:4

C. 1:3

D. 1:2

Correct Response: B. Since *ABCDE* is similar to *PQRST*, corresponding sides of the polygons are proportional. For the polygons in this problem, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2}$. In other words, the length of each side of *PQRST* is twice the length of its corresponding side in *ABCDE*. The area of *ABCDE* can be considered to be equal to the sum of *N* unit squares. The area of *PQRST* is therefore the sum of *N* squares, with each square having sides of length 2. Since the area of *ABCDE* is *N*, the area of *PQRST* is $2 \bullet 2 \bullet N = 4N$. Hence, $\frac{\text{area of } ABCDE}{\text{area of } PQRST} = \frac{N}{4N} = \frac{1}{4}$.

Competency 0012

Understand the principles and properties of coordinate and transformational geometries.

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8. When the lines $y = x + 2$, $y = -4$, and $x = 6$ are graphed on a coordinate grid, what is the area of the polygon enclosed by the three lines?

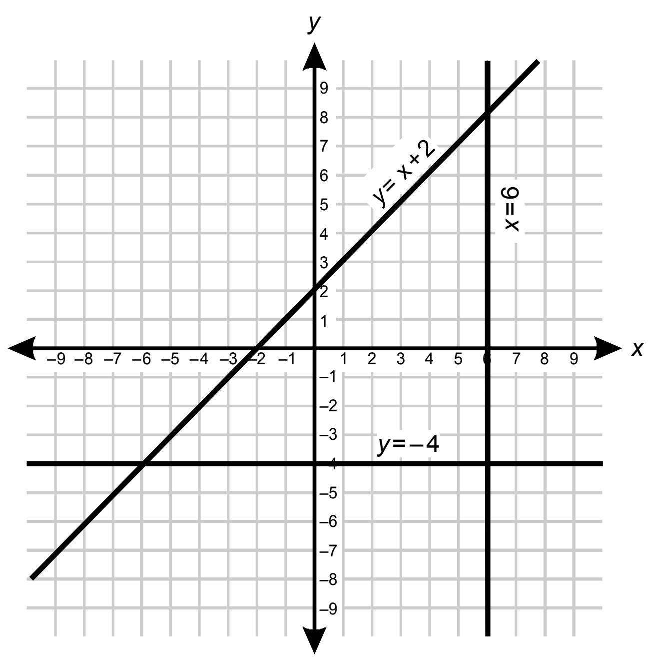
A. 8 square units

B. 32 square units

C. 72 square units

D. 144 square units

Correct Response: C. The first step is to graph the three lines on a coordinate grid.



The polygon enclosed is a triangle with a base of 12 units and a height of 12 units. Since the area of a triangle is $\frac{1}{2}bh$, the area of this triangle is $\frac{1}{2}(12)(12) = 72$ square units.

Competency 0016

Understand the principles of discrete mathematics.

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9. **Use the matrix definitions below to answer the question that follows.**

$R = \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ $T = \begin{bmatrix} 0 & 4 \\ 9 & 2 \end{bmatrix}$

According to the principles of matrix operations, which of the following must be true?

A. *R(ST) = (RS)T*

B. *RS = SR*

C. $R^{-1} = \frac{1}{R}$

D. $(RS)^T = R^T S^T$

Correct Response: A. This is the associative law for multiplication as applied to matrices.

$ST = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 36 & 12 \\ 0 & 8 \end{bmatrix}$, so $R(ST) = \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 36 & 12 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 36 & 12 \\ 180 & 108 \end{bmatrix}$, and

$RS = \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 17 & 20 \end{bmatrix}$, so $(RS)T = \begin{bmatrix} 1 & 4 \\ 17 & 20 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 36 & 12 \\ 180 & 108 \end{bmatrix}$.

Thus, $R(ST) = (RS)T = \begin{bmatrix} 36 & 12 \\ 180 & 108 \end{bmatrix}$.

Competency 0016

Understand the principles of discrete mathematics.

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10. How many five-letter combinations can be formed from the word *teaching*?

    A. $5!$

    B. $\frac{5!}{3!}$

    C. $\frac{8!}{5!}$

    D. $\frac{8!}{5!3!}$

Correct Response: D. A general formula for a combination of n elements taken r at a time is

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!}. $$

In this problem, the eight elements (letters) in the word *teaching* are to be taken five at a time. Therefore, the solution is set up as shown below.

$$ {}_{8}C_{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} $$

11. **Use the information below to complete the exercise that follows.**

A family is going to buy a new air conditioning unit. Brand A costs $2700 to buy and $90 a month to operate. Brand B costs $4200 to buy, but only $70 a month to operate.

Use a system of linear equations to analyze the cost of purchasing and operating each of these two air conditioning units. Your response should show all work and include:

- a particular equation for each unit expressing the total cost of operation as a function of the number of months since the purchase of the unit;
- a sketch of the graphs of the two equations on one set of coordinate axes, with the axes, scales, intercepts, and any calculated points clearly labeled;
- an explanation of the slope and intercepts of each function in the context of the problem situation;
- calculation of the "break-even" point for the two functions, i.e., the number of months for which the total cost of the two units would be the same; and
- advice for the family as to which would be the less expensive option.

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FOR YOUR REFERENCE ONLY—*The constructed-response item is written to assess understanding in Subarea II, Relations, Functions, and Algebra, which consists of the competencies listed below.*

Understand mathematical patterns and use them to solve problems.

Understand the principles and properties of algebraic relations and functions, including inverses and compositions.

Understand the properties of linear functions and relations.

Understand the properties of quadratic and higher-order polynomial functions and relations.

Understand the principles and properties of rational, absolute value, exponential, and logarithmic functions.

A Very Good Response to the Practice Constructed-Response Assignment

- Let A equal the total cost of a Brand A unit and B equal the total cost of a Brand B unit. Let m = the number of months of operation since the purchase of an air conditioner. Then the total cost of a unit is given below.

  Total cost = the initial cost + operating cost per month × number of months

  Brand A
  $A(m) = 2700 + 90m$

  Brand B
  $B(m) = 4200 + 70m$

- These are linear equations. The graph of $A(m)$ has a slope = 90 and a y-intercept = 2700. The graph of $B(m)$ has a slope = 70 and a y-intercept = 4200. To help determine the scale for each axis, solve the system of two equations by first setting them equal and solving for m. This will give the coordinates of the intersection of the two lines.

$$\begin{array}{r} 2700 + 90m = 4200 + 70m \\ \underline{-2700 - 70m = -2700 - 70m} \\ 20m = 1500 \end{array}$$

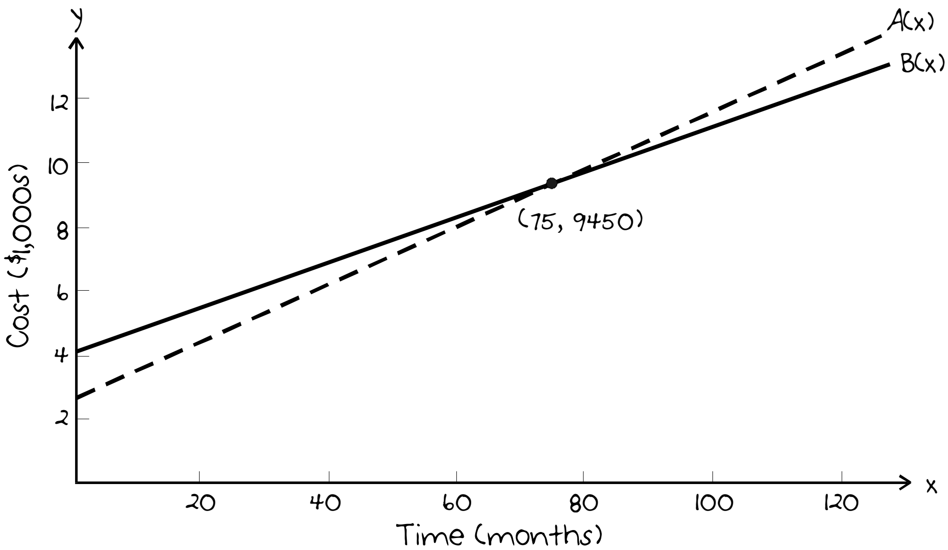
$$m = \frac{1500}{20} = 75$$

  Substitute this value back into one of the equations to find the total cost.
  $A(75) = 2700 + 90(75) = 2700 + 6750 = 9450$
  The lines intersect at (75, 9450).

Since the lines intersect at $m = 75$, extend the scale past this point to get a good representation of the two graphs. For the scale on the x-axis, go from $m = 0$ to $m = 100$ since 100 is a convenient number.
$A(100) = 2700 + 90\ (100) = 11{,}700$
$B(100) = 4200 + 7000 = 11{,}200$

(continued)

For the *y*-axis, go from 0 to 12,000.



- The *y*-intercept of each line represents the initial cost of each unit. The slope of each line represents the monthly operating costs.

- The point of intersection is the "break-even" point. The *x*-coordinate of the point of intersection is the number of months ($m = 75$) when the total costs of both air conditioners is the same. The *y*-coordinate ($A(75) = B(75) = 9450$) is the total cost after 75 months of operation.

- From the graph, the total cost of Brand *A* is less than that of Brand B up to 75 months. After that, the total cost of Brand B is less. The family should buy Brand *A* if they plan to use it for less than 75 months and Brand B if they plan to use it more than 75 months.

CONSTRUCTED-RESPONSE ASSIGNMENT SCORING

All responses to OSAT constructed-response assignments (written and oral) are scored using scoring scales that describe varying levels of performance. These scales were approved by committees of Oklahoma educators who reviewed both the performance characteristics and the scoring scales.

Each response is scored by multiple scorers according to standardized procedures during scoring sessions held immediately after each administration of the CEOE. Scorers with relevant professional backgrounds are oriented to these procedures before the scoring session and are carefully monitored during the scoring sessions.

A response to a constructed-response assignment is designated unscorable if it is blank, not on the assigned topic, illegible or unintelligible, not in the appropriate language, or of insufficient length to score. If you do not provide a scorable response for each constructed-response assignment on your test, you cannot pass the test regardless of your scores on the other section(s) of the test.

Sample Performance Characteristics for Constructed-Response Assignments

PURPOSE	The extent to which the response achieves the purpose of the assignment
SUBJECT MATTER KNOWLEDGE	Accuracy and appropriateness in the application of subject matter knowledge
SUPPORT	Quality and relevance of supporting details
RATIONALE	Soundness of argument and degree of understanding of the subject matter

Sample Scoring Scale for Constructed-Response Assignments

SCORE POINT	SCORE POINT DESCRIPTION
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects a general knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence generally supports the discussion; there are some relevant examples. • The response reflects a general understanding of the topic.
2	**The "2" response reflects a partial knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects little or no knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.
U	**The response is unscorable because it is illegible, not written to the assigned topic, written in a language other than English, or of insufficient length to score.**
B	**There is no response to the assignment.**

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