Certification Examinations
for Oklahoma Educators™

Oklahoma Subject Area Tests™

STUDY GUIDE

011 Advanced Mathematics

Oklahoma Commission
for Teacher Preparation

OK-SG-FLD011-04

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The first two sections of the study guide are available in a separate PDF file. Click the link below to view or print these sections.

Study Guide Introduction and General Information About the Certification Examinations for Oklahoma Educators

FIELD-SPECIFIC INFORMATION

- Test Competencies
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INTRODUCTION

This section includes a list of the test competencies, as well as a set of practice selected-response (multiple-choice) questions and one or more practice constructed-response assignments (if applicable), for the test field included in this study guide.

Test Competencies

The test competencies are broad, conceptual statements that reflect the subject-matter skills, knowledge, and understanding an entry-level educator needs to teach effectively in Oklahoma public schools. The list of test competencies for each test field represents the **only** source of information about what a specific test will cover and therefore should be reviewed carefully.

The descriptive statements that follow the competencies are included to provide examples of possible content covered by each competency. These descriptive statements are neither exhaustive nor exclusionary.

Practice Test Questions

The practice selected-response questions and any practice constructed-response assignments included in this section are designed to give you an introduction to the nature of the questions included in this OSAT test field. The practice test questions represent the various types of questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or to predict your performance on the test as a whole.

To help you prepare for your OSAT, each practice selected-response question in this section is preceded by the competency it measures and followed by a brief explanation of the correct response. On the actual test, the competencies, correct responses, and explanations will **not** be given.

If the test field included in this guide has a constructed-response assignment, a sample response is provided immediately following the practice constructed-response assignment. The sample response in this guide is for illustrative purposes only. Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

A description of the process that is used for scoring the constructed-response assignment is provided in addition to the OSAT performance characteristics and score scale.

When you are finished with the practice test questions, you may wish to go back and review the entire list of test competencies and descriptive statements for your test field.

SUBAREAS:

I. Mathematical Processes and Number Sense
II. Relations, Functions, and Algebra
III. Measurement and Geometry
IV. Trigonometry and Calculus
V. Probability, Statistics, and Discrete Mathematics

SUBAREA I—MATHEMATICAL PROCESSES AND NUMBER SENSE

Competency 0001

Understand mathematical problem solving and the connections between and among the fields of mathematics and other disciplines.

The following topics are examples of content that may be covered under this competency.

Analyze and apply a variety of problem-solving strategies to various contexts.

Select and use appropriate manipulatives and technological tools (e.g., spreadsheets, graphing utilities, statistical packages) to solve problems.

Recognize and apply connections between and among mathematical concepts and other disciplines.

Demonstrate knowledge of the historical development of mathematics, including contributions from diverse cultures.

Competency 0002

Understand the principles and processes of mathematical reasoning.

The following topics are examples of content that may be covered under this competency.

Construct and evaluate mathematical conjectures, arguments, and proofs.

Apply inductive and deductive reasoning to solve problems.

Use counterexamples to formulate and evaluate arguments and disprove suppositions.

Analyze and apply the principle of mathematical induction in proving or disproving arguments.

Competency 0003

Understand and communicate mathematical concepts and symbols.

The following topics are examples of content that may be covered under this competency.

Convert everyday language into mathematical language, notation, and symbols, and vice versa.

Analyze, use, and perform conversions among algebraic, graphic, pictorial, and other modes of presenting and modeling mathematical concepts and relationships.

Deduce the assumptions inherent in a given mathematical statement, expression, or definition.

Evaluate the mathematical thinking and strategies of others.

Competency 0004

Understand number theory and the principles and properties of the real and complex number systems.

The following topics are examples of content that may be covered under this competency.

Apply the properties of integers, fractions, decimals, and percents and their operations in problem-solving situations.

Understand the fundamental principles of number theory (e.g., prime numbers, divisibility).

Analyze and apply algebraic and geometric representations of complex numbers (e.g., polar form, vector form).

Perform and interpret operations on complex numbers (e.g., difference, product, root; geometric interpretation of the sum).

SUBAREA II—RELATIONS, FUNCTIONS, AND ALGEBRA

Competency 0005

Understand the principles and properties of algebraic relations and functions.

The following topics are examples of content that may be covered under this competency.

Distinguish between relations and functions.

Analyze relationships among different representations (e.g., tabular, algebraic, graphic) of relations and functions.

Analyze relations and functions and their graphs in terms of domain, range, intercepts, maxima, and minima.

Determine the effects of transformations [e.g., $f(x + k)$, $kf(x)$] on the graph of a relation or function.

Competency 0006

Understand the principles and properties of linear algebra.

The following topics are examples of content that may be covered under this competency.

Analyze and apply properties involving matrices (e.g., commutative property of addition, associative property of multiplication).

Determine and analyze the inverse and determinant of a matrix.

Represent and solve systems of linear equations using matrices.

Determine and analyze the matrix of a linear transformation.

Competency 0007

Understand the properties of linear functions and relations.

The following topics are examples of content that may be covered under this competency.

Determine and interpret the slope and intercept(s) of a linear equation in mathematical and real-world contexts.

Determine the equation of a line on the basis of different types of information (e.g., two points on the line, the slope and one point on the line).

Model and solve problems involving linear equations and inequalities using algebraic and graphic techniques.

Solve systems of linear equations and inequalities using a variety of techniques (e.g., substitution, graphing).

Competency 0008

Understand the properties of quadratic and higher-order polynomial relations and functions.

The following topics are examples of content that may be covered under this competency.

Analyze relationships among tabular, algebraic, and graphic representations of quadratic and higher-order polynomial functions.

Model and solve problems involving quadratic and higher-order polynomial equations and inequalities using a variety of techniques (e.g., completing the square, factoring, graphing).

Analyze the zeros of quadratic and higher-order polynomial functions and apply their characteristics to solve problems.

Analyze and use the equations and graphs of conic sections.

Competency 0009

Understand the principles and properties of rational, radical, piecewise, and absolute value functions.

The following topics are examples of content that may be covered under this competency.

Manipulate and simplify expressions involving rational, radical, piecewise, and absolute value functions.

Describe and analyze characteristics of rational, radical, piecewise, and absolute value functions and their graphs (e.g., intercepts, asymptotes, domain, range).

Convert between algebraic and graphic representations of rational, radical, piecewise, and absolute value functions.

Model and solve problems involving rational, radical, piecewise, and absolute value equations.

Competency 0010

Understand the principles and properties of exponential and logarithmic functions.

The following topics are examples of content that may be covered under this competency.

Apply the laws of exponents and logarithms to manipulate and simplify expressions.

Analyze and apply the inverse relationship between exponential and logarithmic functions.

Convert algebraic representations of exponential and logarithmic functions into graphic representations, and vice versa.

Model and solve problems involving exponential and logarithmic functions (e.g., compound interest, exponential decay) in mathematical and real-world contexts.

SUBAREA III—MEASUREMENT AND GEOMETRY

Competency 0011

Understand principles and procedures related to measurement.

The following topics are examples of content that may be covered under this competency.

Apply formulas to find measures (e.g., angles, length, perimeter, area, volume) for a variety of two- and three-dimensional figures.

Solve problems involving derived units (e.g., density, pressure, rates of change).

Compare and convert measurements within and between customary and metric measurement systems.

Find angle and arc measures related to circles.

Competency 0012

Understand the principles and properties of Euclidean geometry in two and three dimensions.

The following topics are examples of content that may be covered under this competency.

Use the properties of lines (e.g., parallel, perpendicular) and angles (e.g., supplementary, vertical) to characterize geometric relationships and solve problems.

Apply the principles of similarity and congruence to solve problems involving two- and three-dimensional figures.

Apply the properties of circles (e.g., intersecting chords and secants) and polygons (e.g., numbers and lengths of sides, measures of angles) to analyze and solve problems.

Use definitions, postulates, and theorems of geometry (e.g., Pythagorean theorem) to construct and analyze proofs.

Competency 0013

Understand the principles and properties of coordinate geometry.

The following topics are examples of content that may be covered under this competency.

Apply geometric concepts (e.g., distance, midpoint, slope) to model and solve problems.

Apply the geometric concepts of parallel and perpendicular lines to model and solve problems.

Use two- and three-dimensional coordinate systems to represent and analyze geometric figures.

Represent two- and three-dimensional geometric figures in various coordinate systems (e.g., Cartesian, polar).

Competency 0014

Understand the principles and properties of vector and transformational geometries.

The following topics are examples of content that may be covered under this competency.

Describe the position and movement of objects using vectors.

Model and solve problems involving vector addition and scalar multiplication (e.g., displacement, force).

Analyze and apply geometric transformations (e.g., translations, reflections, dilations, rotations).

Construct and analyze figures using geometric transformations in the coordinate plane (e.g., reflecting across an axis).

SUBAREA IV—TRIGONOMETRY AND CALCULUS

Competency 0015

Understand the principles and properties of and relationships involving trigonometric functions and their graphic representations.

The following topics are examples of content that may be covered under this competency.

Analyze the relationships among right triangle ratios, trigonometric functions, and the unit circle.

Analyze graphs of trigonometric functions in terms of frequency, period, amplitude, and phase shift.

Determine the effects of transformations on the graph of a trigonometric function [e.g., $f(x) = a\sin(bx + c) + d$].

Simplify expressions using trigonometric identities.

Verify trigonometric identities.

Competency 0016

Understand and apply the principles and techniques of trigonometry to model and solve problems.

The following topics are examples of content that may be covered under this competency.

Solve real-world problems using the trigonometry of right triangles.

Apply trigonometric functions and relationships (e.g., law of sines) to model and solve problems involving angles, length, and area.

Model and solve problems involving trigonometric equations and inequalities using algebraic and graphic techniques.

Use trigonometric functions to model periodic phenomena in mathematics and other disciplines.

Competency 0017

Understand the principles and properties of limits, continuity, and average rates of change.

The following topics are examples of content that may be covered under this competency.

Apply the concept of limits to algebraic functions and their graphs.

Analyze and interpret characteristics of functions (e.g., continuity, asymptotes) using the concept of limit.

Recognize and apply the relationship between the slope of a secant line and the derivative of a function.

Solve problems involving average rates of change (e.g., average velocity and acceleration).

Competency 0018

Understand and apply the principles and techniques of differential calculus.

The following topics are examples of content that may be covered under this competency.

Relate the concept of the derivative to instantaneous rate of change and the concept of the slope of the line tangent to a curve.

Find the derivative of a function.

Use the concepts of differential calculus to analyze the graph of a function (e.g., maxima, concavity, points of inflection).

Model and solve real-world problems (e.g., rates of change, optimization, related rates) using differential calculus.

Competency 0019

Understand and apply the principles and techniques of integral calculus.

The following topics are examples of content that may be covered under this competency.

Relate the concept of the integral to the area under a curve.

Find the definite and indefinite integral of a function.

Use integration in problem-solving situations (e.g., area, velocity, volume).

Model and solve problems involving first-order differential equations (e.g., separation of variables, initial value problems).

SUBAREA V—PROBABILITY, STATISTICS, AND DISCRETE MATHEMATICS

Competency 0020

Understand the principles, properties, and techniques of probability.

The following topics are examples of content that may be covered under this competency.

Evaluate descriptions and calculate the probabilities of different kinds of events (e.g., conditional, independent, mutually exclusive).

Solve problems using the techniques of probability (e.g., addition and multiplication rules).

Use and interpret graphic representations of probabilities (e.g., tables, Venn diagrams, tree diagrams, frequency graphs, the normal curve).

Analyze and apply the properties of probability distributions (e.g., binomial, normal) to model and solve problems.

Competency 0021

Understand the principles, properties, and techniques of statistics.

The following topics are examples of content that may be covered under this competency.

Determine random sampling techniques to collect representative data.

Display and use data in a variety of graphic formats (e.g., charts, bar graphs, circle graphs, stem-and-leaf plots, histograms, scatter plots).

Determine, analyze, and interpret measures of central tendency (e.g., mean, median) and dispersion (e.g., standard deviation).

Analyze and interpret statistical measures (e.g., correlation coefficients, confidence intervals, linear regression equations) and make valid inferences and predictions based on the measures.

Competency 0022

Understand the principles of discrete mathematics.

The following topics are examples of content that may be covered under this competency.

Apply various counting strategies (e.g., permutations, combinations) to problem-solving situations.

Analyze recurrence relations (e.g., Fibonacci sequence, triangular numbers) and use them to model and solve problems.

Analyze sequences and series (e.g., arithmetic, geometric) and use them to model and solve problems.

Apply the basic elements of discrete mathematics (e.g., graph theory, linear programming, finite difference methods) to model real-world problems.

Definitions and Formulas for Advanced Mathematics

LOGIC

$a \rightarrow b$	a implies b
$a \leftrightarrow b$	a if and only if b
$a \wedge b$	a and b
$a \vee b$	a or b
$\sim a$	not a
$A \cup B$	A union B
$A \cap B$	A intersect B
$\bar{A}$	complement of A
U	universal set
$\{\ \}$	empty set

ALGEBRA

$i = \sqrt{-1}$	imaginary unit
$\bar{z}$	complex conjugate of z
A^{-1}	inverse of matrix A
$\vec{v}$	vector v

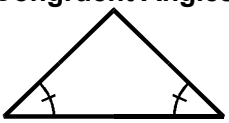
SERIES

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ for } |r| < 1$$

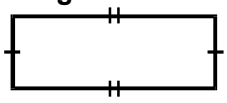
GEOMETRY

$S = 4\pi r^2$	surface area of a sphere
$V = \frac{4}{3}\pi r^3$	volume of a sphere
$\sim$	is similar to
$\cong$	is congruent to

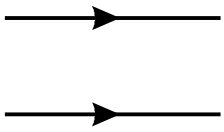
Congruent Angles



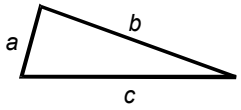
Congruent Sides



Parallel Lines



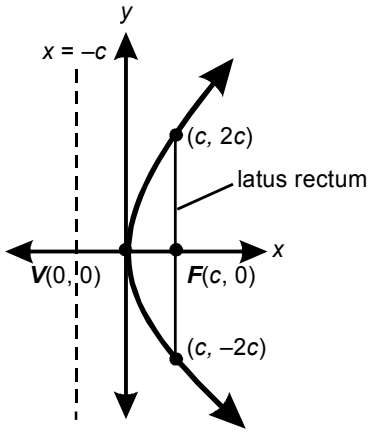
Hero's or Heron's Formula



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

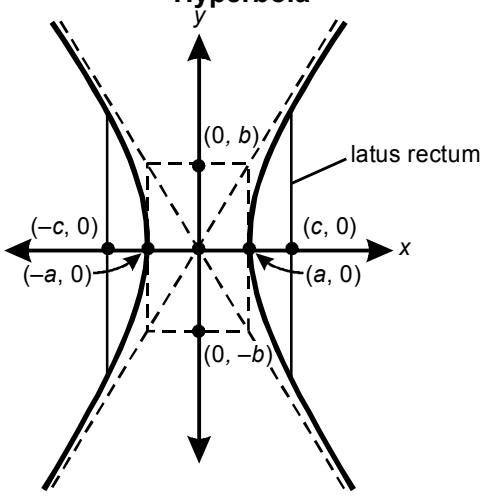
$$\text{where } s = \frac{a+b+c}{2}$$

Parabola



$$(y - k)^2 = 4c(x - h)$$

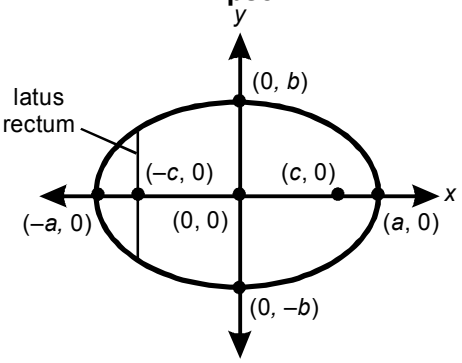
Hyperbola



$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

where $b^2 = c^2 - a^2$

Ellipse



$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

where $c^2 = a^2 - b^2$

Eccentricity of a Conic

$$e = \frac{c}{a}$$

Directrices of a Conic

$$x = \pm\frac{a}{e} = \pm\frac{a^2}{c}$$

TRIGONOMETRY

$$\sin(\theta_1 \pm \theta_2) = \sin\theta_1 \cos\theta_2 \pm \cos\theta_1 \sin\theta_2$$

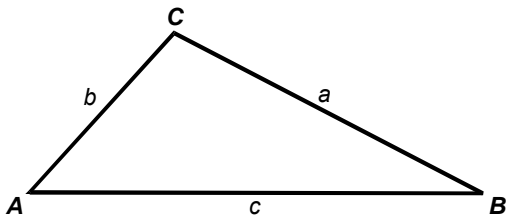
$$\cos(\theta_1 \pm \theta_2) = \cos\theta_1 \cos\theta_2 \mp \sin\theta_1 \sin\theta_2$$

$$\tan(\theta_1 \pm \theta_2) = \frac{\tan\theta_1 \pm \tan\theta_2}{1 \mp \tan\theta_1 \tan\theta_2}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

STATISTICS

$$\text{standard deviation of a sample mean} = \frac{\sigma}{\sqrt{N}}$$

NOTES FOR ADVANCED MATHEMATICS TEST

In this examination, assume all functions are real valued functions unless otherwise noted.

In this examination, diagrams may not be drawn to scale.

In this examination, assume all geometry problems imply the use of Euclidean geometry unless otherwise noted.

All examinees taking the Advanced Mathematics OSAT will be provided with a scientific calculator with functions that include the following: addition, subtraction, multiplication, division, square root, percent, sine, cosine, tangent, exponents, and logarithms. Please see the current CEOE registration bulletin for information regarding the brand and model of calculator that will be supplied. **You may NOT bring your own calculator to the test.**

Practice Selected-Response Questions

Competency 0004

Understand number theory and the principles and properties of the real and complex number systems.

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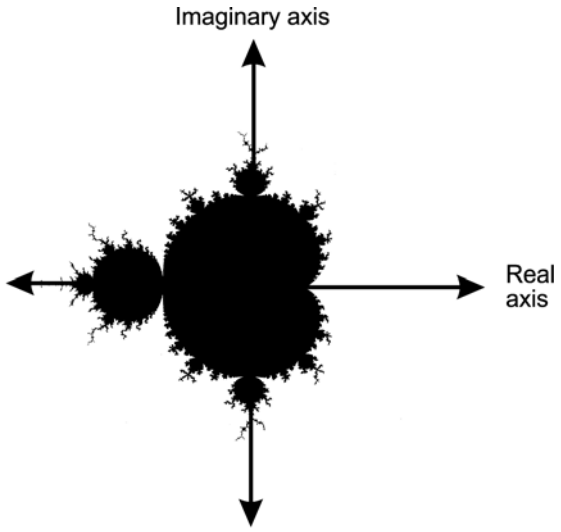
1. **Use the diagram below to answer the question that follows.**



Given that a complex number $z = a + bi$ is an element of the set represented by the shaded region of the complex plane in the diagram above, which of the following must also be an element of the set?

A. $a - bi$

B. $-b + ai$

C. $\frac{1}{a + bi}$

D. $(a + bi)^2$

Correct Response: A. A complex number $z = a + bi$ can be represented in the complex plane as an ordered pair (a, b), where a is the real part and b is the imaginary part of the complex number z. Since the set in the diagram above is symmetric about the x-axis, if (a, b) is an element of the set, then its reflection about the x-axis $(a, -b)$, or $a - bi$, will also be an element of the set.

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2. **Use the problem below to answer the question that follows.**

> A stable contains only people and horses. If there are 94 legs and 27 heads in the stable, how many people and how many horses does it contain?

The problem above can be solved using which of the following matrix equations?

A. $\begin{bmatrix}1 & 1\\ 2 & 4\end{bmatrix}\begin{bmatrix}p\\ h\end{bmatrix}=\begin{bmatrix}27\\ 94\end{bmatrix}$

B. $\begin{bmatrix}1 & 1\\ 2 & 4\end{bmatrix}\begin{bmatrix}94\\ 27\end{bmatrix}=\begin{bmatrix}h\\ p\end{bmatrix}$

C. $\begin{bmatrix}1 & 1\\ 2 & 4\end{bmatrix}\begin{bmatrix}h\\ p\end{bmatrix}=\begin{bmatrix}94\\ 27\end{bmatrix}$

D. $\begin{bmatrix}1 & 1\\ 2 & 4\end{bmatrix}\begin{bmatrix}27\\ 94\end{bmatrix}=\begin{bmatrix}p\\ h\end{bmatrix}$

Correct Response: A. Let p = the number of people and h = the number of horses in the stable. Since people and horses each have one head, $1p + 1h = 27$. Likewise, since people have two legs and horses have four, $2p + 4h = 94$. This system of equations is equivalent to the matrix equation in response A, which can be verified by performing the given matrix multiplication.

Competency 0008

Understand the properties of quadratic and higher-order polynomial relations and functions.

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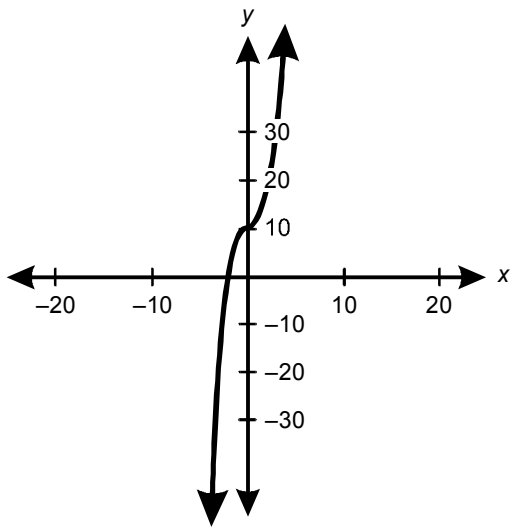
3. **Use the graph below to answer the question that follows.**



The function f(x) = $2x^3 + 10$ is graphed above. Which of the following functions represents a translation of f(x) three units in the positive x-direction?

A. g(x) = $2x^3 + 13$

B. g(x) = $6x^3 + 30$

C. g(x) = $2x^3 – 18x^2 + 54x – 44$

D. g(x) = $2x^3 + 18x^2 + 54x + 64$

Correct Response: C. Given a function f(x), f($x – a$) represents a translation by a units in the positive x-direction. Therefore, given f(x) = $2x^3 + 10$ and $a = 3$, f($x – 3$) = g(x) = $2(x – 3)^3 + 10$ represents a translation by three units in the positive x-direction. Cubing the binomial and simplifying gives g(x) = $2x^3 – 18x^2 + 54x – 44$.

Competency 0012

Understand the principles and properties of Euclidean geometry in two and three dimensions.

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4. In a right triangle, the length of one leg is x, and the length of the second leg is 5 units longer. What is the length of the hypotenuse?

   A. $\sqrt{x^2 + 25}$

   B. $\sqrt{2x^2 + 25}$

   C. $\sqrt{x^2 + 10x + 25}$

   D. $\sqrt{2x^2 + 10x + 25}$

Correct Response: D. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs. Thus, $h^2 = x^2 + (x + 5)^2$ and $h^2 = x^2 + x^2 + 10x + 25$ and finally, $h = \sqrt{2x^2 + 10x + 25}$.

Competency 0013

Understand the principles and properties of coordinate geometry.

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5. A search team has been called in to search for a lost hiker. The hiker was last spotted at coordinates (28, 75) on the search team's map. Since he was last spotted, the hiker could have traveled as much as 15 grid units. Which of the following equations should be used to define the perimeter of the search area?

   A. $(x + 28)^2 - (y + 75)^2 + 15^2 = 0$

   B. $(x - 28)^2 + (y - 75)^2 + 15^2 = 0$

   C. $(x + 28)^2 - (y + 75)^2 - 15^2 = 0$

   D. $(x - 28)^2 + (y - 75)^2 - 15^2 = 0$

Correct Response: D. The hiker could be anywhere within 15 grid units of the point with coordinates (28, 75). This region represents the interior of a circle of radius 15, centered at (28, 75). The perimeter of the search area is given by the equation of the circle. Since a circle is defined to be the set of all points equidistant from a given point (its center), the distance formula can be applied to give $\sqrt{(x-28)^2 + (y-75)^2} = 15$. Squaring both sides of this equation and then subtracting 15^2 from both sides results in response D.

Competency 0014

Understand the principles and properties of vector and transformational geometries.

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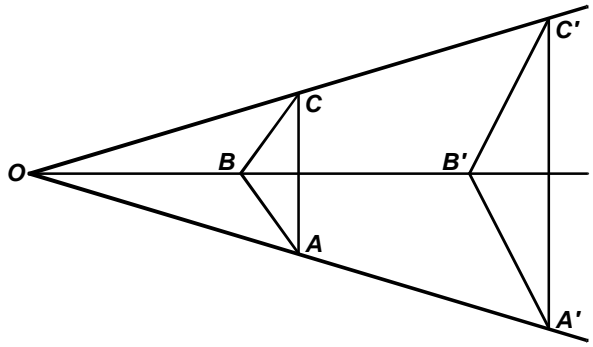
6. **Use the diagram below to answer the question that follows.**



The transformation T of $\triangle ABC$ to $\triangle A'B'C'$ is a dilation with center at O. If the area of $\triangle A'B'C'$ is twice the area of $\triangle ABC$, what is the scale factor of T?

A. $\frac{1}{2}$

B. $\sqrt{2}$

C. 2

D. 4

Correct Response: B. A dilation is a similarity transformation. Therefore, $\triangle ABC \sim \triangle A'B'C'$. If the dilation has scale factor k, then the length of each linear dimension is multiplied by k, and the area (length × length) will be multiplied by k^2. Therefore, (the area of $\triangle A'B'C'$) $= k^2$ (the area of $\triangle ABC$) $= 2$(the area of $\triangle ABC$). Therefore, $k^2 = 2$ and $k = \sqrt{2}$.

Competency 0018

Understand and apply the principles and techniques of differential calculus.

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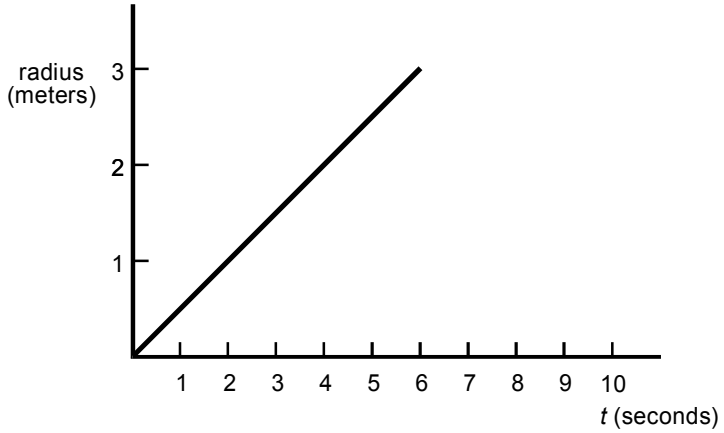
7. **Use the graph below to answer the question that follows.**



The graph above shows the radius of a spherical balloon as a function of time. What is the approximate rate of change in cubic meters per second of the volume of the balloon at t = 4 s?

A. 2.67π

B. 8π

C. 32π

D. 64π

Correct Response: B. The volume of a sphere (from the formulas pages) is $V = \frac{4}{3}\pi r^3$. To determine the rate of change of the volume as a function of time, differentiate the expression implicitly to obtain $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ since the radius r(t) is a function of time. From the graph above, the value of r at $t = 4$ seconds is 2 meters. The derivative of r(t), $\frac{dr}{dt}$, at $t = 4$ seconds is the slope of the line tangent to the curve at $t = 4$, which is $\frac{1}{2}$. Substituting these values into the expression results in $\frac{dV}{dt} = 4\pi(2)^2 \bullet \frac{1}{2} = 8\pi$.

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8. **Use the problem below to answer the question that follows.**

> Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by $y = 2x$, $y = 0$, $x = 3$, and $x = 5$.

Which of the following definite integrals could be evaluated to solve the problem above?

A. $\int_3^5 \pi x^2 \, dx$

B. $\int_3^5 \pi(2x)^2 \, dx$

C. $(5 - 3)\int_0^{2\pi} 2x \, dx$

D. $(5 - 3)\int_0^{2\pi} 2 \sin x \, dx$

Correct Response: B. The volume can be calculated by dividing the solid into a series of cylindrical disks, each having radius $r = f(x)$ and height $h = \Delta x$. Therefore, the volume $V = \lim_{\Delta x \to 0} \sum \pi r^2 h = \int_a^b \pi[f(x)]^2 \, dx$. Given that $f(x) = 2x$, $a = 3$, and $b = 5$, $V = \int_3^5 \pi(2x)^2 \, dx$.

Competency 0020

Understand the principles, properties, and techniques of probability.

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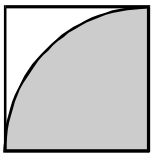
9. **Use the diagram below to answer the question that follows.**



A target consists of a square region in which a quarter circle is drawn and shaded. The radius of the circle is equal to one side of the square. A computer program calculates π by simulating darts being fired randomly at the target. A success is defined as a dart falling within the shaded area. If the computer obtains a value of 3.12 after 1000 shots at the target, how many darts landed in the shaded area? (Assume the probability of hitting the target is 1.)

A. 312

B. 624

C. 750

D. 780

Correct Response: D. Let N represent the number of darts that land in the shaded area. The probability that N darts land in the quarter circle is equal to the ratio of the area of the quarter circle to the area of the square. Let s represent the length of the side of the square. Since the area of a circle is πs^2, $\frac{N}{1000} = \frac{(p/4)s^2}{s^2}$. Substituting 3.12 for the value obtained for π by the computer and simplifying and solving the equation for N results in $N = 780$.

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10. When a computer manufacturing company receives a shipment of computer chips, its quality control inspectors randomly select 10% of the chips in the shipment on which to run tests. If 10% or more of the chips in the sample are defective, the entire shipment is rejected. Which of the following assumptions underlies this decision to test only a sample of chips rather than the entire shipment?

    A. If there are any defects in the shipment, they are most likely to appear in the 10% selected for testing.

    B. The chances of having to reject the load increase as the number of chips increases.

    C. The small sample is fairly representative of the overall shipment.

    D. The probability for human error in the testing procedures increases as the sample size increases.

Correct Response: C. The central limit theorem allows inferences about a population (in this case the shipment of computer chips) to be made from a random sample chosen from the population. These inferences are generally stated in terms of probabilities. In the case given, the quality control engineers are assuming that there is a high probability that the sample represents the population, which is equivalent to response C.

11. **Use the information below to complete the exercise that follows.**

Hours Since Noon	Temperature
2	20°C
3	25°C

The table above shows the outdoor temperature on a given afternoon at two different times.

Use your knowledge of functions to prepare a response in which you model the data as a function of the form $T(x) = \frac{1}{ax + b}$ and as a function of the form $E(x) = cd^x$. In your response:

- find $T(x)$;
- find $E(x)$;
- use each model ($T(x)$ and $E(x)$) to make a prediction for $x = 5$; and
- make a rough sketch of each model on the same coordinate grid, showing the data points as well as the general shape of each function.

Be sure to show your work, including any steps necessary to arrive at your answers.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

FOR YOUR REFERENCE ONLY—*The constructed-response item is written to assess understanding in Subarea II, Relations, Functions, and Algebra, which consists of the competencies listed below.*

Understand the principles and properties of algebraic relations and functions.

Understand the principles and properties of linear algebra.

Understand the properties of linear functions and relations.

Understand the properties of quadratic and higher-order polynomial relations and functions.

Understand the principles and properties of rational, radical, piecewise, and absolute value functions.

Understand the principles and properties of exponential and logarithmic functions.

A Very Good Response to the Practice Constructed-Response Assignment

Using: x = hours since noon $T(x)$ = temperature

$$T(x) = \frac{1}{ax + b}$$

$$T(2) = \frac{1}{2a + b} = 20 \qquad T(3) = \frac{1}{3a + b} = 25$$

$$40a + 20b = 1 \qquad 75a + 25b = 1$$

Solve system of equations:

$$5(40a + 20b = 1) \longrightarrow \quad 200a + 100b = 5$$

$$4(75a + 25b = 1) \longrightarrow \underline{(-)300a + (-)100b = (-)4}$$

$$(-)100a = 1$$

$$a = -0.01$$

Substitute to find b:

$$\frac{1}{2a + b} = 20$$

$$\frac{1}{2(-0.01) + b} = 20$$

$$-0.40 + 20b = 1$$

$$20b = 1.4$$

$$b = 0.07$$

Therefore:

$$T(x) = \frac{1}{-0.01x + 0.07}$$

Using: x = hours since noon $E(x)$ = temperature

$$E(x) = cd^x$$

$$E(2) = cd^2 = 20 \qquad E(3) = cd^3 = 25$$

Solve by dividing:

$$\frac{cd^3}{cd^2} = \frac{25}{20}$$

$$d = 1.25$$

(continued)

Substitute to find c:

$cd^2 = 20$

$c(1.25)^2 = 20$

$c = \dfrac{20}{(1.25)^2}$

$c \approx 12.8$

Therefore:

$E(x) \approx 12.8(1.25)^x$

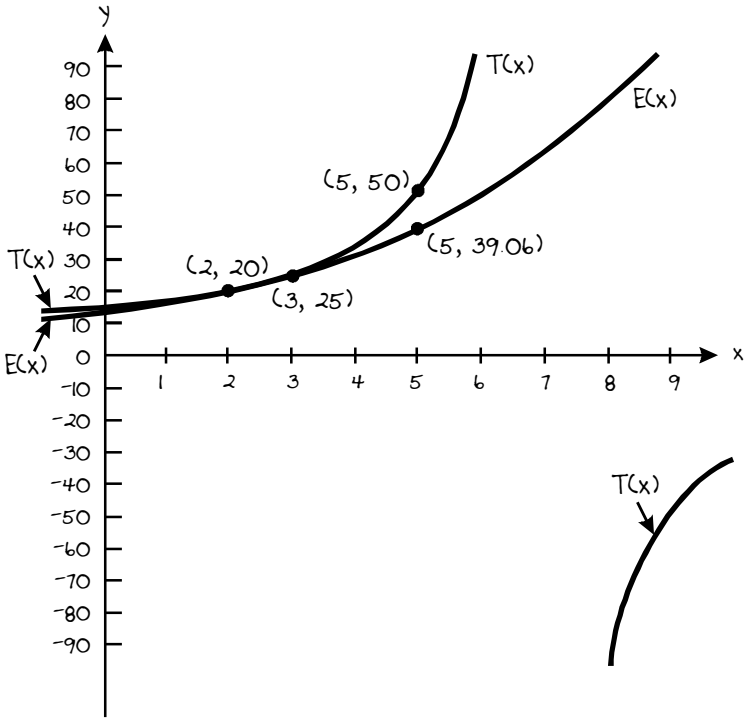
Predict for $x = 5$

$T(x) = \dfrac{1}{-0.01x + 0.07}$ $\qquad$ $E(x) \approx 12.8(1.25)^x$

$T(5) = \dfrac{1}{-0.01(5) + 0.07}$ $\qquad$ $E(5) \approx 12.8(1.25)^5$

$T(5) = 50$ $\qquad$ $E(5) \approx 39.06$

Sketch:



CONSTRUCTED-RESPONSE ASSIGNMENT SCORING

All responses to OSAT constructed-response assignments (written and oral) are scored using scoring scales that describe varying levels of performance. These scales were approved by committees of Oklahoma educators who reviewed both the performance characteristics and the scoring scales.

Each response is scored by multiple scorers according to standardized procedures during scoring sessions held immediately after each administration of the CEOE. Scorers with relevant professional backgrounds are oriented to these procedures before the scoring session and are carefully monitored during the scoring sessions.

A response to a constructed-response assignment is designated unscorable if it is blank, not on the assigned topic, illegible or unintelligible, not in the appropriate language, or of insufficient length to score. If you do not provide a scorable response for each constructed-response assignment on your test, you cannot pass the test regardless of your scores on the other section(s) of the test.

Sample Performance Characteristics for Constructed-Response Assignments

PURPOSE	The extent to which the response achieves the purpose of the assignment
SUBJECT MATTER KNOWLEDGE	Accuracy and appropriateness in the application of subject matter knowledge
SUPPORT	Quality and relevance of supporting details
RATIONALE	Soundness of argument and degree of understanding of the subject matter

Sample Scoring Scale for Constructed-Response Assignments

SCORE POINT	SCORE POINT DESCRIPTION
4	**The "4" response reflects a thorough knowledge and understanding of the subject matter.** • The purpose of the assignment is fully achieved. • There is a substantial, accurate, and appropriate application of subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects an ably reasoned, comprehensive understanding of the topic.
3	**The "3" response reflects a general knowledge and understanding of the subject matter.** • The purpose of the assignment is largely achieved. • There is a generally accurate and appropriate application of subject matter knowledge. • The supporting evidence generally supports the discussion; there are some relevant examples. • The response reflects a general understanding of the topic.
2	**The "2" response reflects a partial knowledge and understanding of the subject matter.** • The purpose of the assignment is partially achieved. • There is a limited, possibly inaccurate or inappropriate application of subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited, poorly reasoned understanding of the topic.
1	**The "1" response reflects little or no knowledge and understanding of the subject matter.** • The purpose of the assignment is not achieved. • There is little or no appropriate or accurate application of subject matter knowledge. • The supporting evidence, if present, is weak; there are few or no relevant examples. • The response reflects little or no reasoning about or understanding of the topic.
U	**The response is unscorable because it is illegible, not written to the assigned topic, written in a language other than English, or of insufficient length to score.**
B	**There is no response to the assignment.**

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