CERTIFICATION EXAMINATIONS FOR OKLAHOMA EDUCATORS™ (CEOE™)

OKLAHOMA SUBJECT AREA TESTS™ (OSAT™)

FIELD 111: ADVANCED MATHEMATICS

TEST FRAMEWORK

September 2016

	Subarea	Range of Competencies
I.	Number Properties and Number Sense	0001–0002
II.	Relations, Functions, and Algebra	0003–0009
III.	Trigonometry and Calculus	0010–0014
IV.	Measurement and Geometry	0015–0017
V.	Probability, Statistics, and Discrete Mathematics	0018–0020
VI.	Pedagogical Content Knowledge	0021

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SUBAREA I—NUMBER PROPERTIES AND NUMBER SENSE

Competency 0001

Apply knowledge of the structure, properties, and operations of the real number system.

The following topics are examples of content that may be covered under this competency.

- Identify the hierarchy of the real number system and its classification into various subsets.
- Analyze the properties of real numbers and their representations (e.g., fractions, radicals, exponents, scientific notation).
- Apply the properties of rational numbers and their operations in problem-solving situations.
- Apply proportional thinking and knowledge of ratios and percentages to represent and solve problems.
- Evaluate mathematical conjectures, arguments, and informal proofs involving numbers, and use counterexamples to evaluate arguments and disprove suppositions.
- Apply the principle of mathematical induction in proving or disproving arguments.

Competency 0002

Apply knowledge of number theory and the complex number system.

The following topics are examples of content that may be covered under this competency.

- Apply the fundamental principles of number theory (e.g., prime numbers, divisibility).
- Apply knowledge of greatest common factors and least common multiples to model and solve problems.
- Analyze algebraic and geometric representations of complex numbers (e.g., polar form, vector form).
- Apply operations on complex numbers (e.g., difference, product, root, geometric interpretation of the sum).

SUBAREA II—RELATIONS, FUNCTIONS, AND ALGEBRA

Competency 0003

Analyze the principles and properties of algebraic relations and functions.

The following topics are examples of content that may be covered under this competency.

- Distinguish between relations and functions.
- Analyze relationships between different representations (e.g., tabular, algebraic, graphic) of relations and functions.
- Analyze relations and functions and their graphs in terms of domain, range, intercepts, maxima, and minima.
- Determine the effects of transformations [e.g., $f(x + k)$, $kf(x)$] on the graph of a relation or function.
- Apply knowledge of technological tools (e.g., spreadsheets, graphing utilities) to analyze functions and solve problems.

Competency 0004

Apply algebraic techniques.

The following topics are examples of content that may be covered under this competency.

- Convert written words and expressions into mathematical notation and symbols, and vice versa.
- Analyze given mathematical statements, expressions, or definitions, and deduce the underlying assumptions.
- Manipulate algebraic expressions and equations (e.g., factoring, simplifying, transforming).
- Justify algebraic techniques using the properties of the real number system, and evaluate mathematical strategies and arguments.

Competency 0005

Analyze the principles and properties of linear algebra.

The following topics are examples of content that may be covered under this competency.

- Apply basic knowledge of the properties of vectors (e.g., components, magnitude) and vector operations to model and solve problems.
- Analyze and apply properties involving matrices (e.g., commutative property of addition, associative property of multiplication).
- Analyze the inverse and determinant of a matrix.
- Use matrices to represent and solve systems of linear equations.
- Analyze the matrix of a linear transformation.

Competency 0006

Apply the properties of linear functions and relations.

The following topics are examples of content that may be covered under this competency.

- Apply the slope and intercept(s) of a linear equation in mathematical and real-world contexts.
- Determine the equation of a line using different types of information (e.g., two points on the line, the slope and one point on the line).
- Model and solve problems involving linear equations and inequalities using algebraic and graphic techniques.
- Solve systems of linear equations and inequalities using a variety of techniques (e.g., substitution, graphing).

Competency 0007

Analyze the properties of quadratic and higher-order polynomial relations and functions.

The following topics are examples of content that may be covered under this competency.

- Analyze relationships between tabular, algebraic, and graphic representations of quadratic and higher-order polynomial functions.
- Model and solve problems involving quadratic and higher-order polynomial equations and inequalities using a variety of techniques (e.g., completing the square, factoring, graphing).
- Analyze the zeros of quadratic and higher-order polynomial functions and apply their characteristics to solve problems.
- Analyze the equations and graphs of conic sections.

Competency 0008

Apply the principles and properties of rational, radical, piecewise, and absolute value functions.

The following topics are examples of content that may be covered under this competency.

- Manipulate and simplify expressions involving rational, radical, piecewise, and absolute value functions.
- Analyze characteristics of rational, radical, piecewise, and absolute value functions and their graphs (e.g., intercepts, asymptotes, domain, range).
- Convert between algebraic and graphic representations of rational, radical, piecewise, and absolute value functions.
- Model and solve problems involving rational, radical, piecewise, and absolute value equations.

Competency 0009

Apply the principles and properties of exponential and logarithmic functions.

The following topics are examples of content that may be covered under this competency.

- Apply the properties of exponents and logarithms to manipulate expressions and solve equations.
- Analyze and apply the inverse relationship between exponential and logarithmic functions.
- Convert between algebraic and graphic representations of exponential and logarithmic functions.
- Model and solve problems involving exponential and logarithmic functions (e.g., compound interest, exponential decay) in mathematical and real-world contexts.

SUBAREA III—TRIGONOMETRY AND CALCULUS

Competency 0010

Analyze the principles and properties of and relationships involving trigonometric functions and their graphic representations.

The following topics are examples of content that may be covered under this competency.

- Analyze the relationships between right triangle ratios, trigonometric functions, and the unit circle.
- Analyze graphs of trigonometric functions in terms of frequency, period, amplitude, and shifts (vertical and phase).
- Analyze the effects of transformations on the graph of a trigonometric function [e.g., $f(x) = a \sin(bx + c) + d$].
- Simplify expressions using trigonometric identities.
- Verify trigonometric identities.

Competency 0011

Apply the principles and techniques of trigonometry to model and solve problems.

The following topics are examples of content that may be covered under this competency.

- Solve mathematical and real-world problems using the trigonometry of right triangles.
- Apply trigonometric functions and relationships (e.g., law of sines) to model and solve problems involving angles, length, and area.
- Model and solve problems involving trigonometric equations and inequalities using algebraic and graphic techniques.
- Use trigonometric functions to model periodic phenomena in mathematics and other disciplines.

Competency 0012

Apply the principles and properties of limits, continuity, and average rates of change.

The following topics are examples of content that may be covered under this competency.

- Apply the concept of limits to algebraic functions and their graphs.
- Analyze the characteristics of functions using the concept of limit (e.g., continuity, asymptotes).
- Apply the relationship between the slope of a secant line and the derivative of a function.
- Solve problems involving average rates of change (e.g., average velocity and acceleration).

Competency 0013

Apply the principles and techniques of differential calculus.

The following topics are examples of content that may be covered under this competency.

- Relate the concept of the derivative to instantaneous rate of change and the concept of the slope of the tangent line to a curve.
- Find the derivative of a function.
- Use the concepts of differential calculus to analyze the graph of a function (e.g., extrema, concavity, points of inflection).
- Model and solve real-world problems using differential calculus (e.g., rates of change, optimization, related rates).

Competency 0014

Apply the principles and techniques of integral calculus.

The following topics are examples of content that may be covered under this competency.

- Relate the concept of the integral to the area under a curve.
- Find the definite and indefinite integral of a function.
- Apply integration to solve problems (e.g., area, velocity, volume, distance).
- Model and solve problems involving first-order differential equations (e.g., separation of variables, initial value problems).

SUBAREA IV—MEASUREMENT AND GEOMETRY

Competency 0015

Apply principles and procedures related to measurement.

The following topics are examples of content that may be covered under this competency.

- Apply formulas to find measures (e.g., angles, length, perimeter, area, volume) for a variety of two- and three-dimensional figures.
- Solve problems involving derived units (e.g., density, pressure, rates of change).
- Convert measurements within and between various measurement systems.
- Find angle and arc measures related to circles.

Competency 0016

Apply the principles and properties of Euclidean geometry in two and three dimensions.

The following topics are examples of content that may be covered under this competency.

- Characterize geometric relationships and solve problems using the properties of lines (e.g., parallel, perpendicular) and angles (e.g., supplementary, vertical).
- Apply the principles of similarity and congruence to solve problems involving two- and three-dimensional figures.
- Apply the properties of circles (e.g., intersecting chords and secants) and polygons (e.g., number and length of sides, measure of angles) to analyze and solve problems.
- Use definitions, postulates, and theorems of geometry (e.g., Pythagorean theorem) to evaluate mathematical conjectures and arguments, and construct and analyze proofs.

Competency 0017

Apply the principles and properties of coordinate geometry.

The following topics are examples of content that may be covered under this competency.

- Apply geometric concepts (e.g., distance, midpoint, slope) to model and solve problems.
- Apply the geometric concepts of parallel and perpendicular lines to model and solve problems.
- Analyze and apply geometric transformations (e.g., translations, reflections, dilations, rotations).
- Represent two- and three-dimensional geometric figures in various coordinate systems (e.g., Cartesian, polar).
- Apply knowledge of connections between mathematical concepts in different fields (e.g., algebra, coordinate geometry, probability).

SUBAREA V—PROBABILITY, STATISTICS, AND DISCRETE MATHEMATICS

Competency 0018

Apply the principles, properties, and techniques of probability.

The following topics are examples of content that may be covered under this competency.

- Evaluate descriptions and calculate the probabilities of different kinds of events (e.g., conditional, independent, mutually exclusive).
- Solve problems using the techniques of probability (e.g., addition and multiplication rules).
- Interpret graphic representations of probabilities (e.g., tables, Venn diagrams, tree diagrams, frequency graphs, the normal curve).
- Analyze and apply the properties of probability distributions (e.g., binomial, normal) to model and solve problems.

Competency 0019

Apply the principles, properties, and techniques of statistics.

The following topics are examples of content that may be covered under this competency.

- Apply random sampling techniques to collect representative data.
- Interpret data in a variety of graphic formats (e.g., charts, bar graphs, circle graphs, stem-and-leaf plots, histograms, scatter plots).
- Analyze and apply measures of central tendency (e.g., mean, median) and dispersion (e.g., standard deviation).
- Analyze statistical measures (e.g., correlation coefficients, confidence intervals, linear regression equations) and make valid inferences and predictions that are based on the measures.

Competency 0020

Apply the principles of discrete mathematics.

The following topics are examples of content that may be covered under this competency.

- Apply various counting strategies (e.g., permutations, combinations) to problem-solving situations.
- Analyze and apply recurrence relations (e.g., Fibonacci sequence, triangular numbers) to model and solve problems.
- Analyze and apply sequences and series (e.g., arithmetic, geometric) to model and solve problems.
- Apply the basic elements of discrete mathematics (e.g., graph theory, linear programming, finite difference methods) to model real-world problems.

SUBAREA VI—PEDAGOGICAL CONTENT KNOWLEDGE

Competency 0021

Analyze a lesson plan for a given learning standard in the Oklahoma Academic Standards for Mathematics, including examples of student work and/or assessments, and describe subsequent activities that address student needs on the basis of your analysis.

The following topics are examples of content that may be covered under this competency.

- Apply knowledge of standards-based learning goals for mathematical content and processes.
- Analyze assessment results of student learning and/or samples of student work for a particular lesson in mathematics, citing specific evidence from the exhibits that identifies a significant mathematical strength as well as a significant area of need shown by the student or students.
- Describe an appropriate instructional strategy or intervention that would help the student or students improve in the identified area of need.
- Describe how your analysis of assessment data and/or student work can be used to inform future instruction with respect to this content area and the development and reinforcement of sound mathematical practice.